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# MATHEMATICS AND PSYCHOLOGY.*

By C. C. Grove.

In mental vision, upon the wall of a great hall, I see a clock with weights and pendulum exposed. An electric light is dependent before it, causing dark clear shadows on the wall—of the weights like inverted pine-cones, of the pendulum like a wicked one fleeing with none pursuing, faster and still faster, ever fainter, to the other end of the hall.

It often happens in human affairs that men get their attention focused on the scurrying shadow and not on the stately rhythmic swing of the pendulum, upon the unreal instead of the real. Thus it was, several years ago, that a friend, an able teacher of secondary mathematics, came asking about certain declarations then recently made about the value and future of mathematics in the secondary schools. There was real concern in his questioning. Instinctively I thought of one Demetrius, the silversmith, of Ephesus.

At that time, as now also, I thought there was no occasion for concern to those who are of the way, to those who have breathed in the spirit of mathematics, whose vision is clear to see the true normal swing of the pendulum. I further believe now, as then, that the permanent appreciation and conception of our subject on the part of the masses rests with, and shall remain in the hands of those who teach mathematics.

It is for us to examine ourselves to see whether we have received the spirit of mathematics. If we have not, we shall be troubled by many things; if we have, we shall not be alarmed by the statements of those who have not. Until we recognize the spirit of mathematics as the sun about which the whole system revolves, a hundred questions will harass and perplex us—questions as to transfer, as to parts of that formal outer husk and hard shell of mathematics—the mere formal processes and rules—that will transfer and be useful in the curricula of the schools of various types. Too much teaching of mathematics has been a "pouring-in process" and a drill in formal operations, instead of a presentation of the method of inquiry into any subject and the means of making intelligible a large body of the world's intellectual activity. Mathematics is not "simply deductive logic" but claims sisterhood with the humanities and is quite at home in their company. Not all interpretative courses need be confined to history and philosophy. Read addresses by Professor Cassius J. Keyser in SCIENCE, Nov. 12, '15, *The Human Significance of Mathematics*, and Mar. 26, '15, *Mathematical Instruction.*

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* Under the above broad title, so stated for brevity on the program of the annual meeting of the Association at Drexel Institute, November 27, 1915, there was presented a paper that dealt with two separate lines of contact of the departments of psychology and mathematics. The first part considered an attitude towards the so-called "movement against mathematics in the secondary schools" which was in large part initiated on account of conclusions drawn from investigations made by experimental psychologists. This part follows. The second part, to be published in the near future, deals with the mathematics employed by psychologists, and points out the foundations on which this mathematical work rests.

Teachers who get no further in their understanding and appreciation of mathematics than the formal stage, just indicated, are in Milton's words "blind mouths" whose

> lean and flashy songs
> Grate on their scrannel pipes of wretched straw,

while

> The hungry sheep look up and are not fed.

They are neither bishops nor shepherds. Rather we look to the teacher who feels with Browning the vernal throb of new abounding life and says,

> Grow old along with me!
> The best is yet to be,
> The last of life, for which the first was made.

Should not the heart beat once, "How good to live and learn"? These teachers have spiritual children. The spiritually dead cannot beget spiritual children. The letter killeth, the spirit giveth life.

We must recognize mathematics as "the ideal and norm of all careful thinking" (G. Stanley Hall), that only through mathematical reasoning can we approach the ultimate reality of the eternal world, for, as Bergson has said, "the more science digs into things the more it tends to mathematics." Yet we must also see that the spirit and method of mathematics pervade the humblest human activities.

The maid asked my wife, "Does Mr. Grove wish to alarm the clock?" I have seen a person turn a profile photograph edgewise to get the full-face view of the friend represented. I have heard a person, telling of an unsuccessful hunt for a certain number on a street in Philadelphia, say, "One went on each side of the street and we walked for blocks without finding it." Someone said "Poor horse, one would think he could not fall with all that harness on." You laugh, yet all these except the first were spoken by educated people. Quietly examine how many people act and speak as though they were the children of Epimetheus instead of Prometheus; how many cannot follow directions, read a recipe; how many attempting to do something, miss the very heart and purpose, do the manifest external processes, and so leave it half done, which is frequently worse than not done at all. Observe how difficult it is for a physician to get the essentials to diagnose a case. Many looking for peas cannot see pumpkins all around. Thus we might multiply illustrations of how people very generally do not analyze the situations of life and do not act intelligently. Dr. Katherine B. Davis, Commissioner of Corrections of New York, in an address said, "If only the schools would teach children to appreciate and distinguish cause and effect, they would do more than by teaching geography, history, etc."

Now it seems highly reasonable that a course that will present to children conditions clearly stated and will demand careful accurate thinking on the same, with a clear correct statement in good English of the analysis of the conditions and the logical result, will furnish a drill, if kept within the capacity and experience of the child, that will not permit such loose superficial thinking, or lack of thinking. Such a course, in my experience, is a properly administered course in mental arithmetic. One of the first fruits of the Pestalozzian movement and the beginning of true reform in mathematical teaching in the United

States was Warren Colburn's "Intellectual Arithmetic" (1821). I received an incalculable blessing by such a course from Brooks's "Mental Arithmetic." Into some such courses there came to be introduced so many special features that the real issue was overlooked. Even where this was not the case, teachers often found it very difficult to conduct the course properly because they had not caught the spirit of it. Then it was easy to say, "We are spending too much time on arithmetic." The statement in itself was very true but the remedy altogether wrong; they discontinued mental arithmetic and continued the formal "written arithmetic" under teachers often that had to have the "Key" as their indispensable "vade mecum."

Experience tells me that some such careful training in analysis, coupled with the demand for accurate statement in good English, is an essential, alike for men and women, for the intelligent conduct of life in any sphere. The habit of analyzing problems of life, of probing to the real issues, of demanding sound not merely plausible reasons, the habit of thinking and not seeking intellectual salvation without it—these, I say, are fostered directly by such study and the question of transfer does not arise, for the process of thinking is identical in each. The conditions in mathematical problems are usually clearly and fully given; in life the conditions must be determined by experience and a form of scientific imagination. The reasoning is the same however. Thinking is thinking; and the great majority of people try to avoid more than is absolutely necessary of it because they have not been trained to it nor learned the joyful mastery it brings to those who have the habit. This habit is not to be formed by the study of mathematics alone; yet through such study the habit, we feel, can be most readily and thoroughly formed, because here possibly better than in any other study can be demanded "the patience to be thorough, the concentration to understand, and the persistence to grasp and to apply" (President Butler's Annual Report, 1913, p. 35), and failure can be most easily detected.

[In these and other words, in the oral delivery of the paper, I tried to show that the chief concern of mathematicians as to "the movement against mathematics in the schools" need be only to continue quietly and more earnestly to lead the increasing number of good students into a realization of the *spirit* of mathematics, to train teachers who, with due acknowledgments to Robert Browning, regard what they have to offer and present, "not merely as a substitute for a good cigar or a game of cards," nor yet as a great spoonful of the treacle of Squeers, that must be swallowed willy-nilly. Such teachers will consider that they fail of their mission unless they teach *more* than mathematics, unless mathematics becomes for them the best means of inspiring students with a love for truth and with a joy in seeing a body of truth being built up into a great cathedral-like structure as the embodiment and expression of the life and character of the builder. Such teaching is its own motivation for student and teacher alike. Like the thoroughbred, it has the spur in its veins. If you do not believe it, try it or go sit under a teacher who will demonstrate it to you.

Then were pointed out some sorts of the barnacles encrusting our educational bark:

The large proportion of young unmarried teachers who have no idea of following teaching as a profession, as recently shown by Mr. Harold W. Foght, of the U. S. Bureau of Education, Bulletin 49, 1914. These naturally lower the spirit of teaching for they bring little or no spirit to it.

A certain class of educators who, considering the proposition that there is no transfer of mathematical ability to have been proven by psychologists, are doing much harm by loudly advocating a "practical vocational" training, which turns out undisciplined, uncultured artisans, whose "primary object" is to have "good producing ability as a marketable asset," and also by getting the child to elect to place himself into a veritable caste system before he has seen any more of the glory of living than they choose to see. They do not recognize that at any time the industrial equilibrium may be upset and it may be necessary to bring the same body of workers, through long costly training, into successful readjustment.

Also the all too common practice of teaching down to children, as opposed to letting them come to enjoy the fellowship and companionship of their elders. Look up the life of John Ruskin, of Oscar Wilde, of Carl Witte, of Horace Mann, and read the memorial to the late Dr. James M. Greenwood in *School and Society* of March 20, 1915, pp. 409, 410. Parents and teachers alike have much to learn of the inestimable value of the intimate friendly association of young and old.

Another form of teaching down to children is repeated drilling on what the child comprehends at the first presentation, until the child fairly groans within himself to be freed from that dead body of truth, loosed from the hold of the whirlpool into which he has come.

Such factors influencing education throughout, barnacles retarding its progress, are not recognized, their effects are charged to the *subject* of mathematics instead of to the spirit in which it is taught, and the ground for criticism is apparently much increased.

Finally there has been strongly claimed for the conclusions drawn from so-called mental tests that these rest on a sure scientific, mathematical basis. Many, not knowing statistical methods, have then transferred to the interpretations of the results of these tests the universal feeling of certitude or confidence in mathematics. Here, you will note, is a case of actual transfer far more easily proven than that mathematical training does not transfer. May it not be that the contention is that of the fly as to the elephant in the introduction to the "Budget of Paradoxes," and that all that is needed is to await the coming or birth of more flies when the matter will settle itself? In fact it seems that many without the psychological ability, insight and experience of their master may bring even the good accomplished into disrepute and doubt by rushing into formal applications of his methods. The great increase in carefulness in expression in recent years may be seen by comparing, *e..g.*, Professor E. L. Thorndike's "Educational Psychology," 1913, Vol. II, p. 425, with earlier statements running back to 1903.

Thus we come to our initial statement that the alarmists have their attention on the scurrying shadow and not on the real situation. We want to get, keep, or continue disseminating the spirit of mathematics according to our individual positions. We need to be brought by the criticism to serious self-examination of ourselves and our teaching, and to bring the latter to its proper, high plane. The criticism will then, like a summer's cloud, soon disappear.

The psychologists, I know, are glad to learn where they have gone astray, and especially to learn the fundamental assumptions made in the development of the statistical methods they have employed; also the applicability or the non-applicability of the methods to their problems. This you will find in the forthcoming second part of the paper.

(*To be concluded in next issue.*)

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